Forum for Electromagnetic Research Methods and Application Technologies (FERMAT)

Solution of Electrically Large Scattering problems using the Characteristic Basis Function Method

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Abstract: In this presentation, parallel multilevel characteristic basis function method (MLCBFM) has been hybridized with the randomized pseudo-skeleton approximation method (RSPA) and randomized singular value decomposition (rSVD) method, for the analysis of the scattering from electrically large rough surfaces at low grazing angles. MLCBFM defines the Characteristic Basis Functions (CBFs) on a larger domain and thereby achieves a higher compression rate. The reduced matrix has a much smaller size compared to that of the original system, which enables us to use a direct solver, rather than an iterative one. The RSPA algorithm accelerates the generation of the reduced matrix while the rSVD expedites the generation of CBFs. The hybrid method is found to be both accurate and efficient, and we use it in this work to investigate the problem of bistatic scattering from Gaussian and exponential rough surfaces at low grazing angles.

Keywords: Characteristic basis Functions (CBFs), Rough Surface, Scattering, rSVD, RSPA.

References:

[1] I.MFuks and A. G. Voronovich, “Wave diffraction by rough interfaces in an arbitrary plane-layered medium,” Waves in Random Media, vol.10, pp. 253–272, 2000.
[2] N. Pinel, J. T. Johnson and C. Bourlier, "A Geometrical Optics Model of Three Dimensional Scattering From a Rough Layer With Two Rough Surfaces," Antennas and Propagation, IEEE Transactions on, vol. 58, pp. 809-816, 2010.
[3] M. El-Shenawee, "Polarimetric scattering from two-layered two-dimensional random rough surfaces with and without buried objects," Geoscience and Remote Sensing, IEEE Transactions on, vol. 42, pp. 67- 76, 2004.
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[6] C. Li, G. Bianconi, R. Mittra, and G. Zhu, “Parallelized Multilevel Characteristic Basis function Method (MLCBFM) combined with Adaptive Cross Approximation

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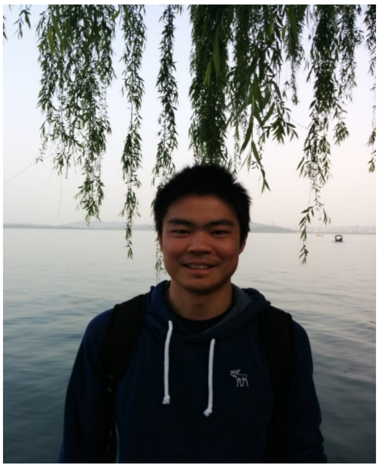
[7] X. Zhu and W. Lin, "Randomised pseudo-skeleton approximation and its application in electromagnetics," Electronics Letters, vol. 47, no. 10, p. 1, 2011.



Chao Li received Ph.D. degree in radio physics from Wuhan University, Wuhan, in 2014. From September 2013 to April 2014, he was a visiting scholar with the Pennsylvania State University. From April 2015 to May 2016, he was a Postdoctoral Research Fellow working in the Electromagnetic Communication (EMC) laboratory with the University of Central Florida. Since May 2016, he has been a lecturer with the University of Jinan, Jinan, Shandong. His research interest includes Method of Moments based fast algorithms, electromagnetic scattering, antenna synthesis, ground penetrating radar and imaging techniques.

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He is a Life Fellow of the IEEE, a Past-President of AP-S, and he has served as the Editor of the Transactions of the Antennas and Propagation Society. He won the Guggenheim Fellowship Award in 1965, the IEEE Centennial Medal in 1984, and the IEEE Millennium medal in 2000. Other honors include the IEEE/AP-S Distinguished Achievement Award in 2002, the Chen-To Tai Education Award in 2004 and the IEEE Electromagnetics Award in 2006, and the IEEE James H. Mulligan Award in 2011.

Recently he founded the e-Journal FERMAT (www.e-fermat.org) and has been serving as the co-editor-in-chief of the same. Dr. Mittra is a Principal Scientist and President of RM Associates, a consulting company founded in 1980, which provides services to industrial and governmental organizations, both in the U.S. and abroad.

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OUTLINE

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- **MULTILEVEL CBFM**
- **RANDOMIZED SINGULAR VALUE DECOMPOSITION**
- **NUMERICAL RESULTS**
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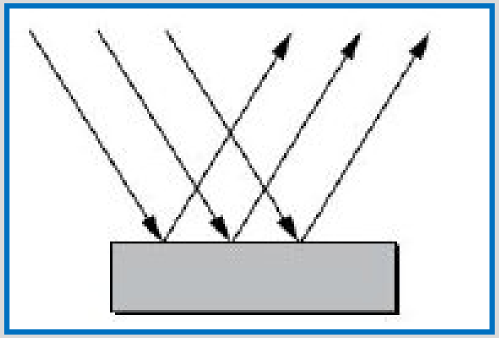
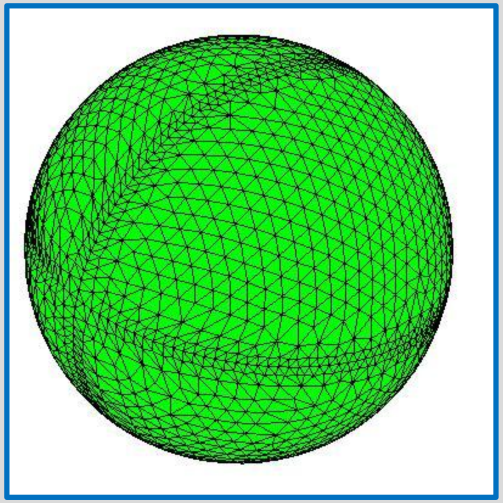


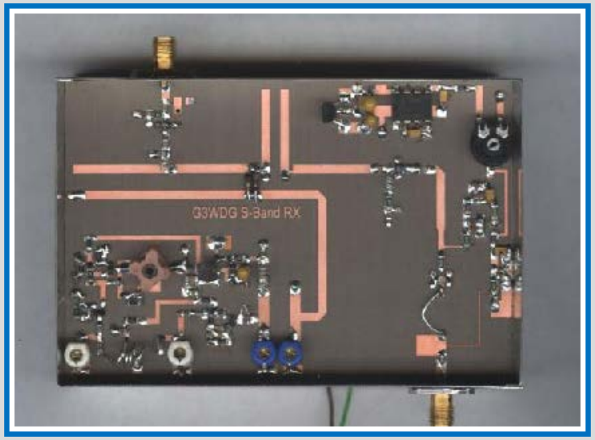


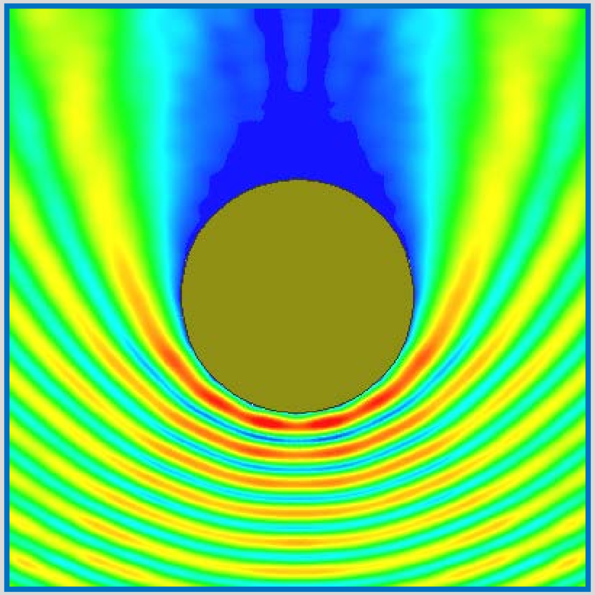
MOTIVATION (1)

- Higher compression rate can be achieved by using larger domain in CBFM[R1]. However, the increased degree of freedom (DOF) in large domain makes the generation of the Characteristic Basis Functions (CBFs) very time- and memory-consuming. The reasons are: 1) Impedance matrix pertaining to each domain should be calculated; 2) Singular Value Decomposition (SVD) is adopted to remove the redundancy between the CBFs.

- To mitigate such problem, Multilevel CBFM (MLCBFM) has been proposed.

[R1] J. Laviada, F. Las-Heras, M. R. Pino and R. Mittra, "Solution of Electrically Large Problems With Multilevel Characteristic Basis Functions," *Antennas and Propagation, IEEE Transactions on*, vol. 57, no. 10, pp. 3189-3198, October 2009



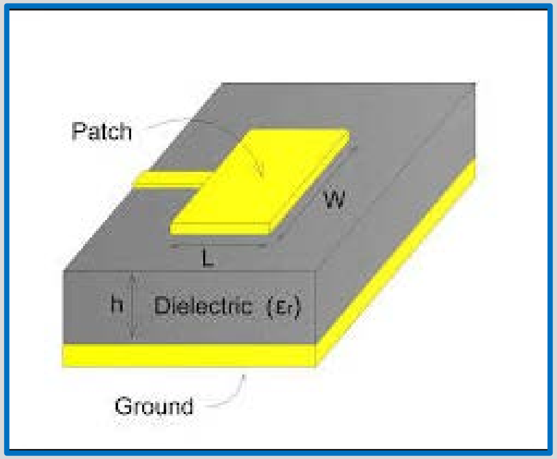


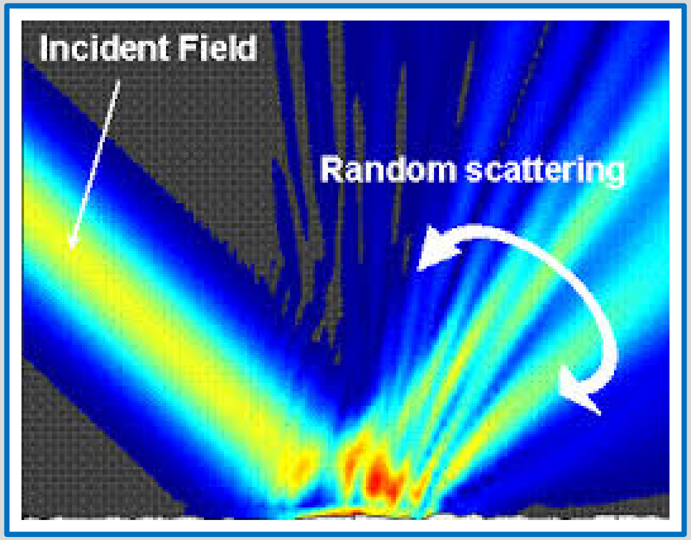
MOTIVATION (2)

❖Similar to MLCBFM, a hybrid approach, namely the hybrid CBFM/ACA/UV method, was developed in our recent work to enable the usage of large-size blocks [R2].

❖ However, the increased number of unknowns in large-size blocks leads to an increase in the computational cost, because it becomes necessary to carry out the Singular Value Decomposition (SVD) to remove the redundancy between the CBFs and, as a consequence, the overhead for generating the CBFs is dominated by the computational cost of the SVD operation.

[R2] Chao, L. and R. Mittra, "Accelerating solution of rough surface scattering problems by using the UV technique in conjunction with the Characteristic Basis Function Method and the Adaptive Cross Approximation " IEEE International Symposium on Antennas and Propagation and USNC-URSI Radio Science Meeting, 2015.

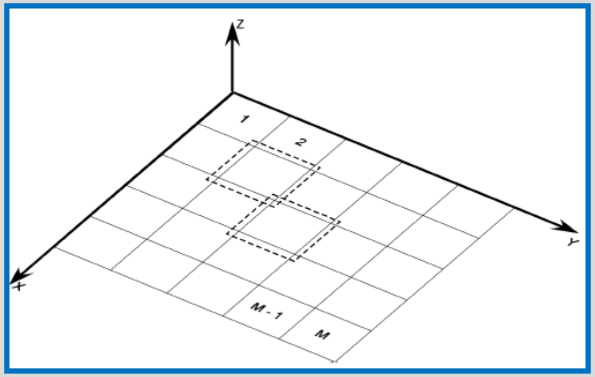




CHARACTERISTIC BASIS FUNCTION METHOD

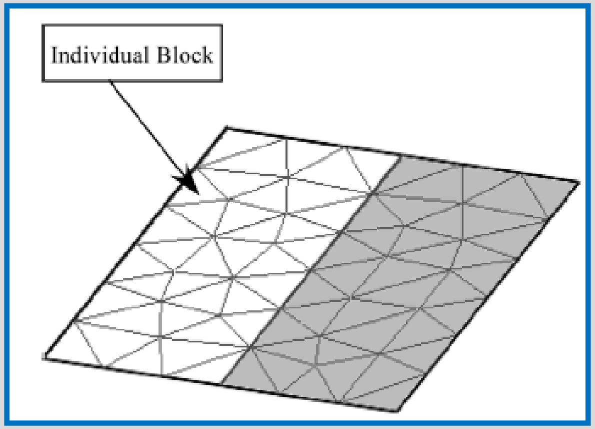
Step - I: The first step in **CBFM** is to divide the problem in smaller regions with respect to the original geometry, called “blocks” [R_4]

- The induced currents in each blocks are represented by using a type of high-level basis function defined **Primary Basis Functions**



Step - II: The **Primary Basis Functions** are evaluated by analyzing each block in the absence of the others:

$$\underline{\underline{Z}}_{ii} \cdot \underline{P}_{i,n} = \underline{V}_{i,n} \qquad \begin{matrix} i = 1,2,\cdots,M \\ n = 1,2,\cdots,N \end{matrix}$$



[R3] E. Lucente, A. Monorchio and R. Mittra, "Parameterization of the tapered incident wave for numerical simulation of electromagnetic scattering from rough surface," *Antennas and Propagation, IEEE Transactions on*, vol. 56, no. 4, pp. 999-1006, April 2008

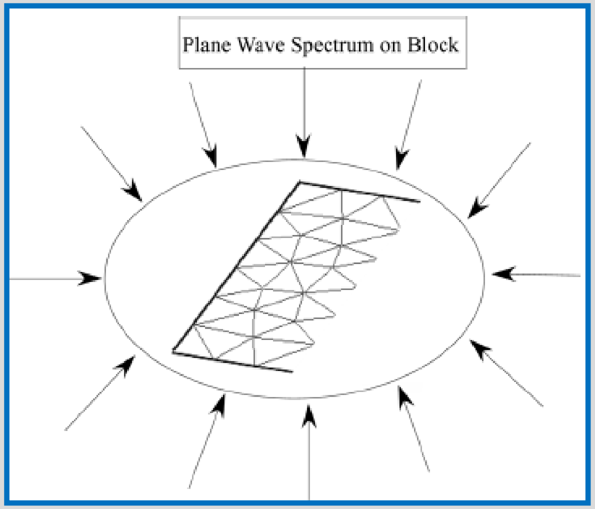
CHARACTERISTIC BASIS FUNCTION METHOD

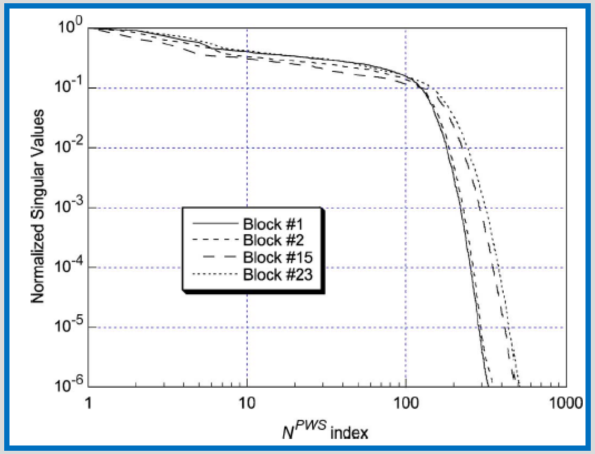
- Each block is excited by using a **Spectrum Of Plane Waves (SPWs)** intentionally overestimating the **Degrees Of Freedoms (DOFs)** of the problem

Step - III: After generating the CBFs, a thresholding procedure based on the a **Singular Value Decomposition (SVD)** approach, can be employed in order to reduce the number of CBFs and the reduced matrix size

Step - IV: The final current distribution ***J*** induced on the scatterer can be expressed as a linear combination of the **CBFs** provided by the **SVD** procedure (**$\underline{J}_{n,m}$**):

$$\underline{J} = \sum_{n=1}^{N_1} \alpha_{1,n} \underline{J}_{1,n} + \sum_{n=1}^{N_2} \alpha_{2,n} \underline{J}_{2,n} + \cdots + \sum_{n=1}^{N_M} \alpha_{M,n} \underline{J}_{M,n}$$





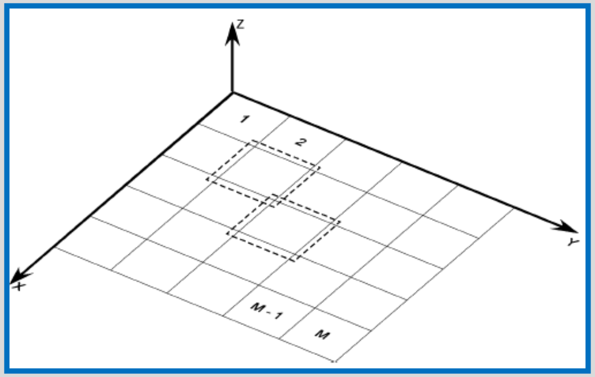
CHARACTERISTIC BASIS FUNCTION METHOD

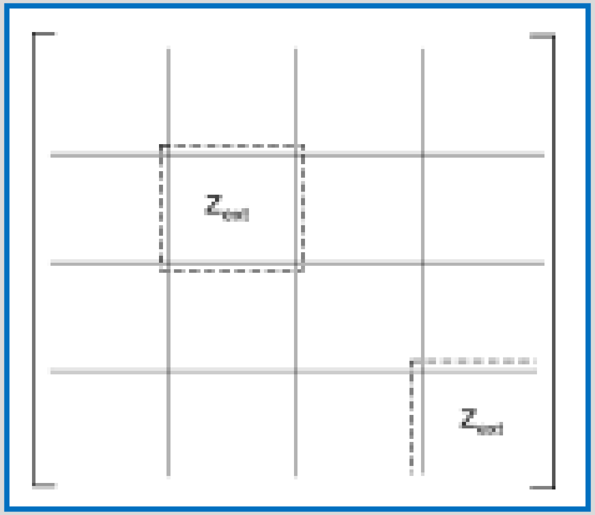
Step – V: The final step to be performed is the generation of the **Reduced matrix *Z^r^*** that can be accomplished by applying the Galerkin testing procedure employing the CBFs as testing functions

$$\underline{\underline{Z}}^{r} \cdot \underline{\alpha} = \underline{V}^{r} \qquad \underline{\underline{Z}}^{r} = \underline{J}^{T} \cdot \underline{\underline{Z}} \cdot \underline{J} \qquad \underline{V}^{r} = \underline{J}^{T} \cdot \underline{V}$$

Z: MoM impedance matrix
J: matrix comprising all the CBFs after the SVD procedure
V: RHS vector related to the real problem excitations

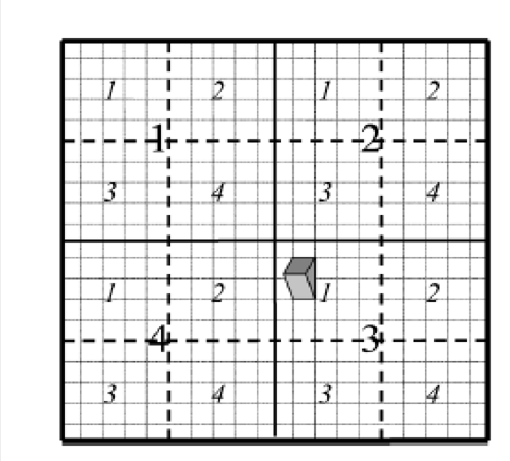
❖ The CBFM leads to a reduced matrix, which is much smaller than the original one, and this obviates the need for an iterative solution of problems requiring a large number of unknowns

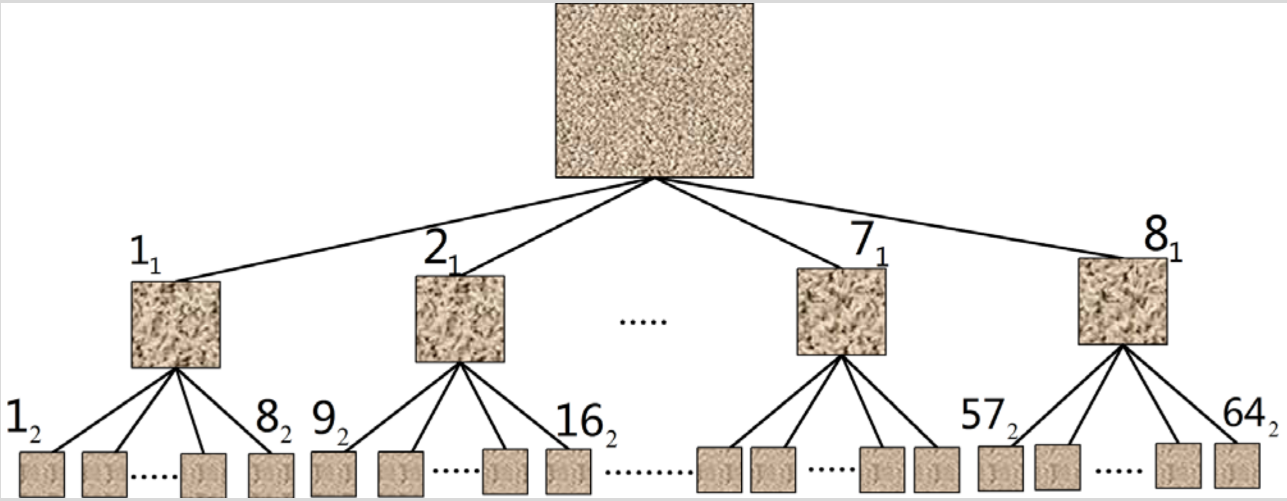




Hybrid CBFM/UV/ACA Method

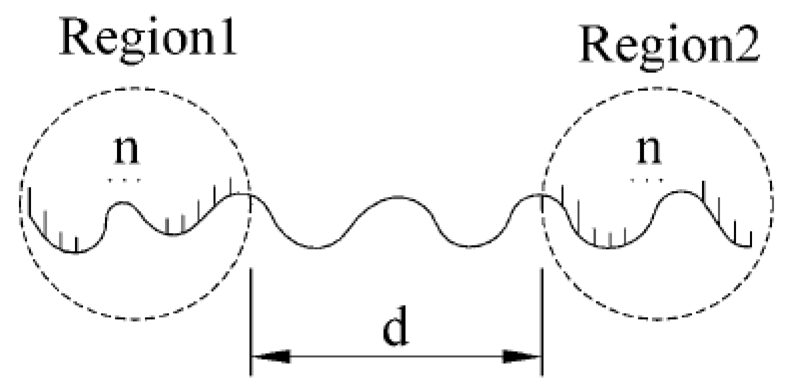
- a hybrid approach, namely the hybrid CBFM/ACA/UV method, was developed in our recent work to address the same issue. By using UV technique, the computational time and memory complexity required by the matrix filling process are decreased from O(B $\bar{N}_{RWG}^{2}$)to O(B $\bar{N}_{RWG}$ log $\bar{N}_{RWG}$), where B is the number of blocks and $\bar{N}_{RWG}$ is the average number of RWG in each block. Comparing to MLCBFM, hybrid CBFM/ACA/UV has the advantage that it does not need to generate either the CBFs or the reduced matrix in the lower level (two-level domain decomposition policy is normally adopted for MLCBFM).





UV decomposition method: for scalar problem

For scalar wave scattering, the rows and columns can be sampled uniformly in the source and field regions to select the most important interpolation points, and the method is called **coarse-coarse sampling** and proved to be quite efficiently*.

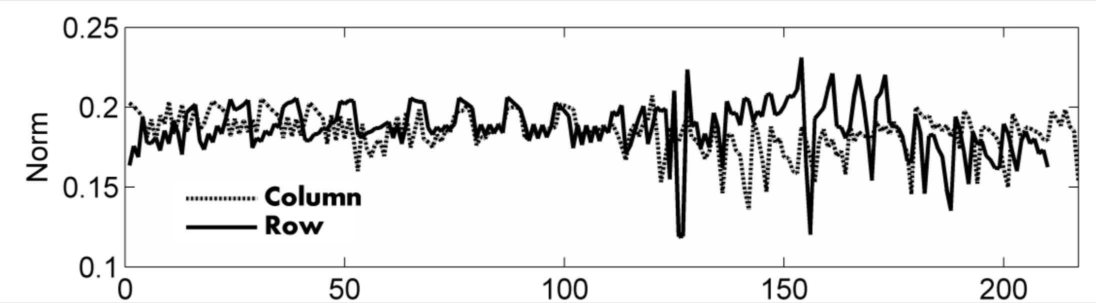


Two well-separated regions with coarse-coarse sampling

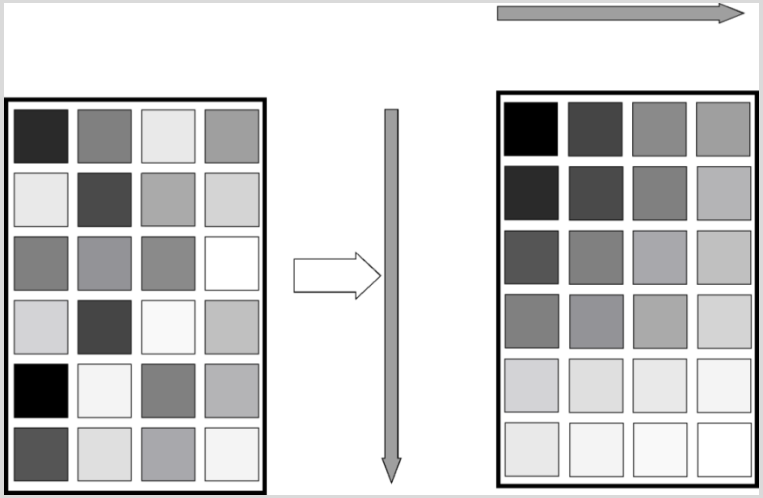
However, for vector wave scattering problem, where the RWG vectors basis function is employed, the EM interaction is not only related to the space distance but also affected by the current direction of the RWG function. Consequently, the matrix element varies abruptly and the smooth characteristic of the matrix elements that the original UV method demands doesn't exist .

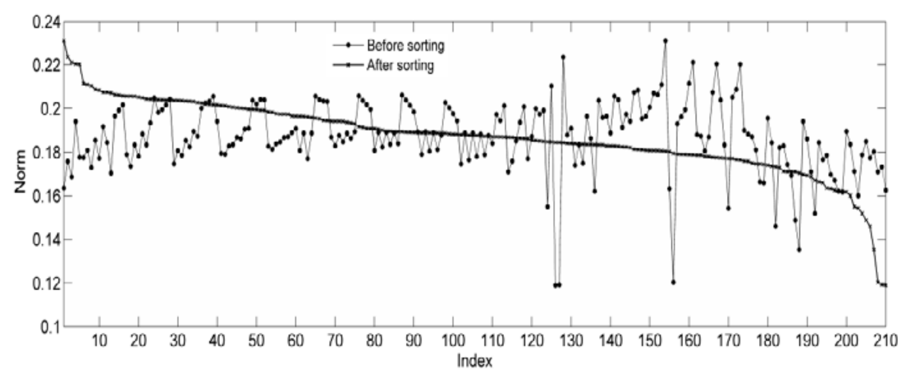
*L.Tsang, Q. Li, and P. Xu, "Wave scattering with the UV multilevel partitioning method: 2. Three-dimensional problem of nonpenetrable surface scattering," Radio Science, vol. 39, RS5011, 2004.

New sampling method for vector problem



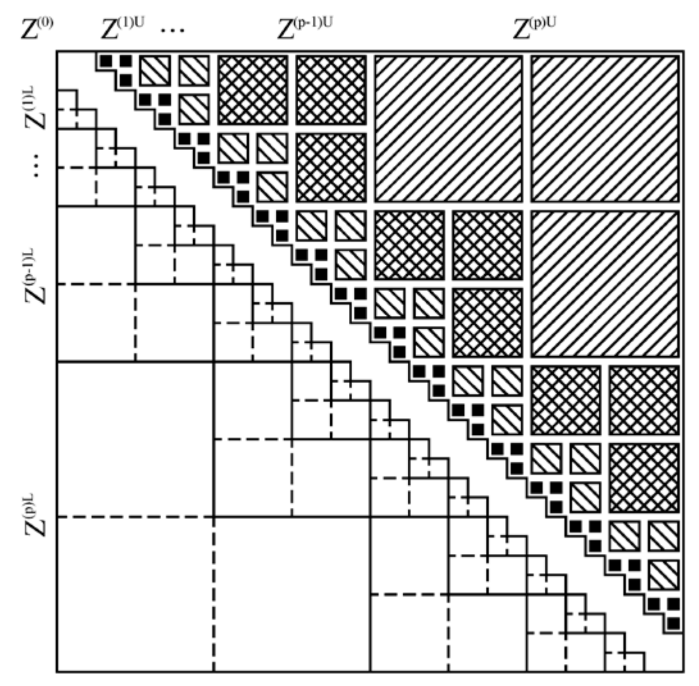
The EM interaction for a far interaction submatrix from the 3-D cube scattering problem. The distribution is disordered and oscillatory according to the natural space ordering.





Re-ordeing of the EM interaction. It is observed, after sorting, the norm varies monotonically according to the new ordering, i.e. the interaction between the source and field regions are re-ordered by their strength from the strongest to weakest, form the point view of the physics.

Acceleration of the matrix vector multiplication



Matrix Vector Multiplication

$$\overline{\overline{Z}} = \sum_{i=1}^{N_n} (\overline{\overline{Z}}_{near})_i + \sum_{j=1}^{N_f} (\overline{\overline{Z}}_{far})_j$$

$$(\overline{\overline{Z}}{}_{far}^{sub})_{m\times n} = \overline{\overline{U}}_{m\times r}\,\overline{\overline{V}}_{r\times n}, \quad r << \min(m,n)$$

$$\overline{\overline{Z}}\,\overline{I}^{(n_p)} = (\sum_{i=1}^{N_n} (\overline{\overline{Z}}_{near})_i)\,\overline{I}^{(n_p)} + (\sum_{j=1}^{N_f} (\overline{\overline{Z}}_{far})_j)\,\overline{I}^{(n_p)}$$

$$(\overline{\overline{Z}}{}_{far}^{sub})_{m\times n}\,\overset{\simeq}{I}{}_{n\times 1}^{(n_p)} = \overline{\overline{U}}_{m\times r}\,(\overline{\overline{V}}_{r\times n}\,\overset{\simeq}{I}{}_{n\times 1}^{(n_p)})$$

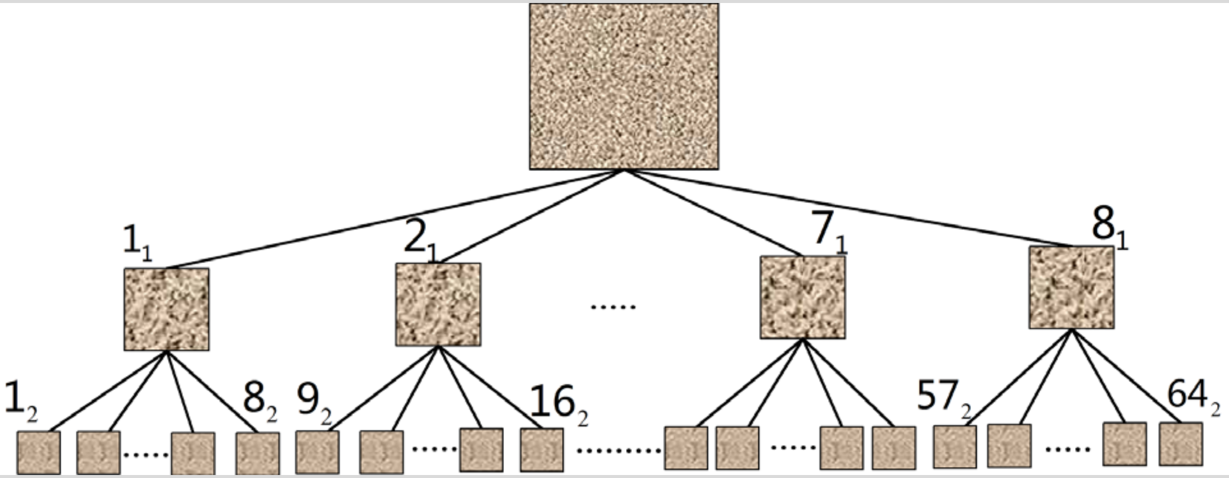
Memory requirement and Computation complexity

$$m\times n \Longrightarrow r\times(m+n)$$

$$O(N^2) \Longrightarrow O(N\log N)$$

Hybrid CBFM/UV/ACA Method

❖ The UV technique is used to accelerate the fill-in of the submatrices pertaining to blocks in **level 1**

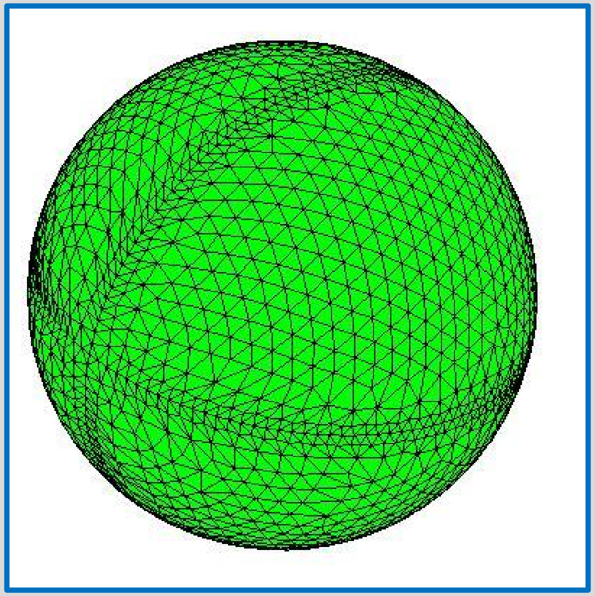


❖The ACA method is used to accelerate the generation of the reduced matrix in **level 1**

Comparing to MLCBFM, hybrid CBFM/ACA/UV has the advantage that it does not need to generate either the CBFs or the reduced matrix in the lower level

CODE VALIDATION

- The developed code has been validated by analyzing the canonical example of a Perfect Electric Conductor (PEC) sphere



- The results provided by the Hybrid CBFM/ACA/UV have been compared with the conventional CBFM ones and with the Mie solution

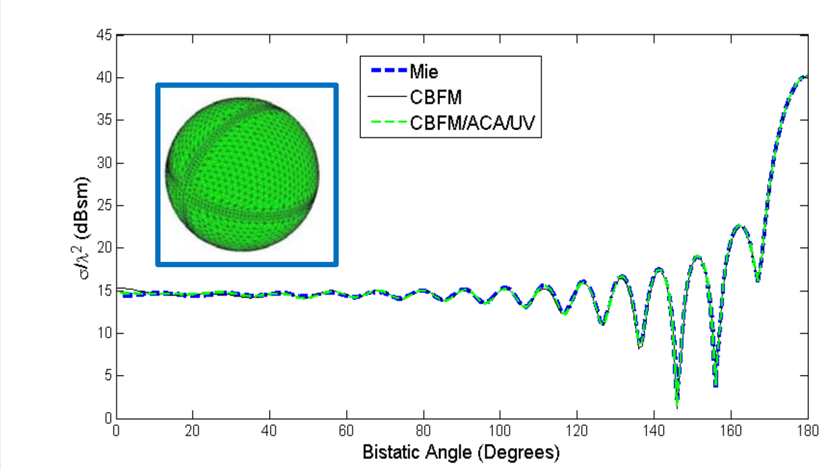
- The code accuracy have been tested by using the reported Relative Error definition

- Sphere radius: 3.0m
- Frequency: 300MHz
- Number of RWGs: 52,023
- SVD threshold: $1e^{-3}$
- ACA threshold: $1e^{-3}$

$$Rel.\,Error = 100 * \sum_{n} \frac{\left| RCS_{MCBFM,n} - RCS_{Mie,n} \right|}{\left| RCS_{Mie,n} \right|}$$

CODE VALIDATION

Bistatic RCS:



Performance summary:

	CBFM	CBFM/ACA/UV
Number of blocks	64	16
Dimensions of the reduced matrix	8598	2932
Solution time(s):	551	137

$$\mathit{Relative\ Error}\left(\%\right) = 0.3327$$

❖ Very good agreement has been achieved between the proposed approaches

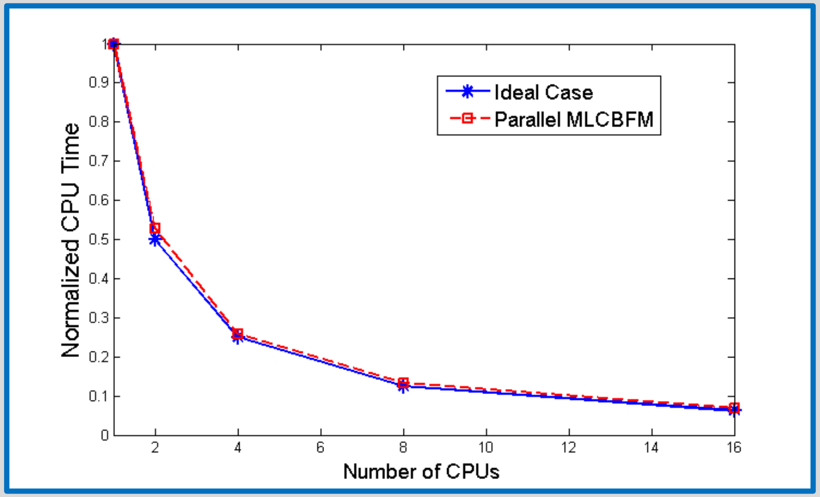
- Sphere radius: 3.0m
- Frequency: 300MHz
- Number of RWGs: 52,023
- SVD threshold: 1e-3
- ACA threshold: 1e-3

CODE VALIDATION

❖ We have also tested the numerical efficiency of the parallel algorithm by progressively increasing the number of CPUs employed to solve the sphere problem

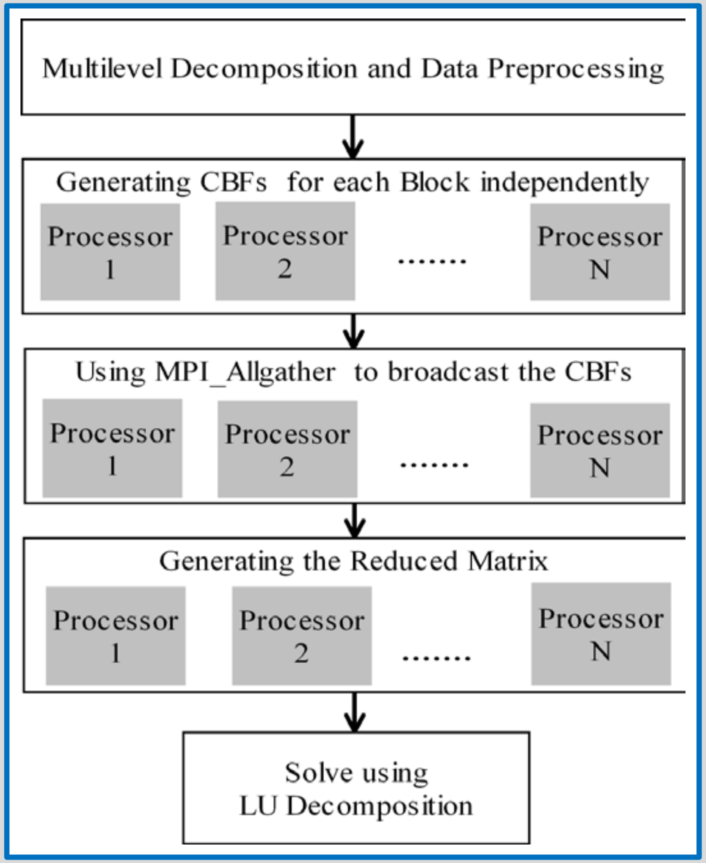
❖ The obtained efficiency has been compared with respect to the ideal case where the solution time scales as *1/n*

CPU Time:



❖ The parallelized CBFM algorithm scales quite well

Parallel Implementation:



Multilevel Decomposition and Data Preprocessing

Generating CBFs for each Block independently

Processor 1 | Processor 2 | ……. | Processor N

Using MPI_Allgather to broadcast the CBFs

Processor 1 | Processor 2 | ……. | Processor N

Generating the Reduced Matrix

Processor 1 | Processor 2 | ……. | Processor N

Solve using LU Decomposition

THE randomized Singular Value Decomposition

❖ The **randomized Singular Value Decomposition (rSVD)** has been implemented in order to efficiently generate the CBFs of the large blocks in MLCBFM

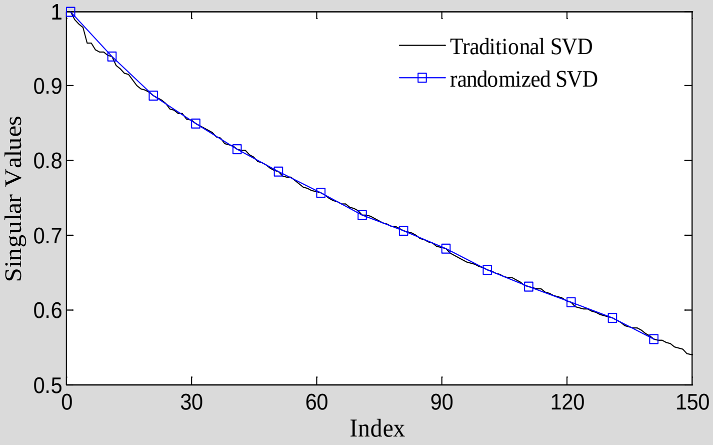
The rSVD begins by randomly projecting the original matrix to a lower-dimension matrix, while the range of the original matrix is asymptotically kept intact. The projected matrix, which is much smaller, is then factorized by using a full-matrix decomposition such as the SVD, after which the resulting singular vectors are back-projected to the original space. The algorithm can be readily implemented by using the following steps:

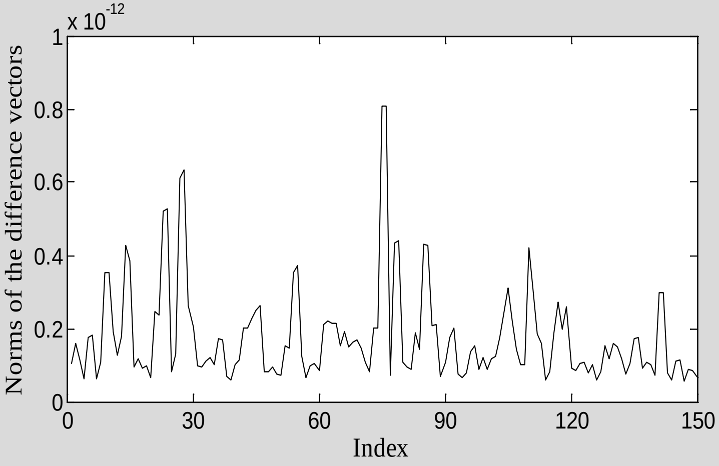
1. Generate a collection of l random vectors of length N (from a Gaussian distribution) and arrange them in columns to from a rectangular matrix R with a dimension of $N \times l$.
2. Compute an $M \times l$ sample matrix $Y=AR$. The column vectors form a basis for the range of A.
3. From an $M \times l$ matrix Q whose columns form an orthonormal basis for the columns of matrix Y. Then $A=QQ^{*}A$.
4. Form the $l \times N$ “small” matrix $B= Q^{*}A$. Then $A=QB$.
5. Form the SVD of B (cheap since B is “small”), $B= \overline{U}\Sigma V^{T}$.
6. Form $U=Q\overline{U}$.

THE randomized Singular Value Decomposition

The accuracy of the rSVD is considered by the test case of scattering by a 4λ × 4λ rectangular plate, illuminated by a spectrum of plane waves incident from 0° $<\vartheta\leq 180°$ and 0° $<\Phi\leq 360°$, with $N_{\vartheta}=18$ and $N_{\Phi}=36$. We begin by calculating the coefficients of pre-SVD CBFs (characteristic basis functions) by using the Method of Moments (MoM), and then arrange the CBFs in columns to form a matrix J^{CBFs}, which has a dimension of 1568 × 1296. Next, traditional SVD and rSVD are performed on J^{CBFs}. The post-SVD CBFs are derived and denoted as U^{SVD} and U^{rSVD} respectively. Following this we define a difference matrix U^{diff} as

$$U^{\text{diff}} = U^{\text{SVD}} - U^{\text{rSVD}}$$





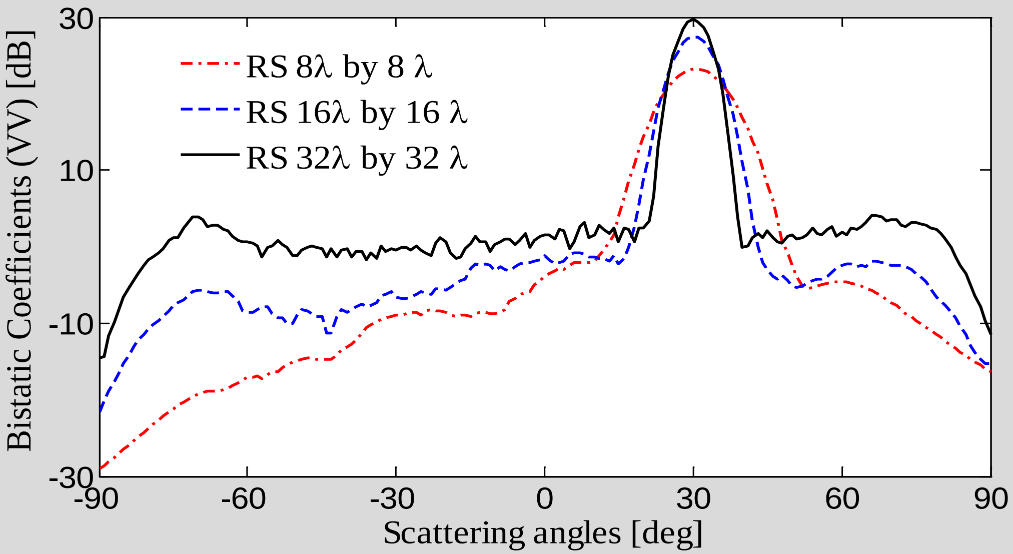
THE randomized Singular Value Decomposition

Four pre-SVD CBFs matrices are generated by enlarging the dimension of a rectangular plate to show the efficiency of the rSVD. The dimensions are 1568 × 1296, 2720 × 1296, 4608 × 1296 and 8368 × 1296. The number of random vectors *l* is chosen to be 550. The CPU times of the SVD and rSVD are shown in Table 1, from which we can see that rSVD is much more efficient than SVD, especially when the dimensions of the object are large.

Matrix	**1**	**2**	**3**	**4**
SVD	0.74s	7.63s	58.62s	488.23s
rSVD	0.15s	0.3s	0.94s	3.46s

Numerical Result-Rough Surface Scattering

Two-dimensional Perfect Electric Conductor (PEC) rough surfaces are analysed by using MLCBFM. Three surfaces whose sizes are 8λ×8λ(case 1), 16λ×16λ(case 2) and 32λ×32λ(case 3) are considered. The root mean square (RMS) height is chosen to be 0.168λ and the correlation length is assumed to be 10 times the RMS height. 100 realizations are implemented to derive the statistical results.



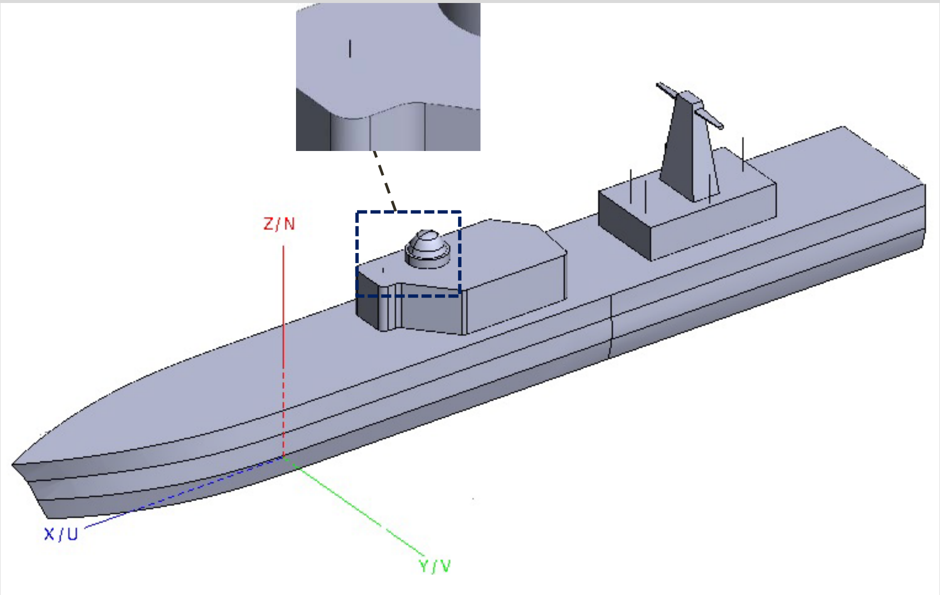
The angular widths of coherent bistatic scattering coefficients become smaller with an increase of the size of the surface

Numerical Result-Rough Surface Scattering

As discussed above, we desire to keep the block sizes large enough to gain a high compression rate. Two different block sizes are adopted and we also use SVD to obtain the results, instead of the rSVD, for the sake of comparison. The computation time for a single realization is shown in Table 2. It is observed that when the block size is chosen to be large, using the rSVD in the MLCBFM algorithm results in significant time-saving. For the rough surface with dimensions of 32λ×32λ, the total time saved for the derivation of the statistical result was on the order of 81 hours.

Block size	**Case 1**		**Case 2**		**Case 3**	
	SVD	rSVD	SVD	rSVD	SVD	rSVD
2λ	288s	243s	×		×	
4λ	251s	192s	1905s	1581s	8460s	6976s
8λ	×		1678s	942s	7264s	4345s

Numerical Result-Radiation Problem

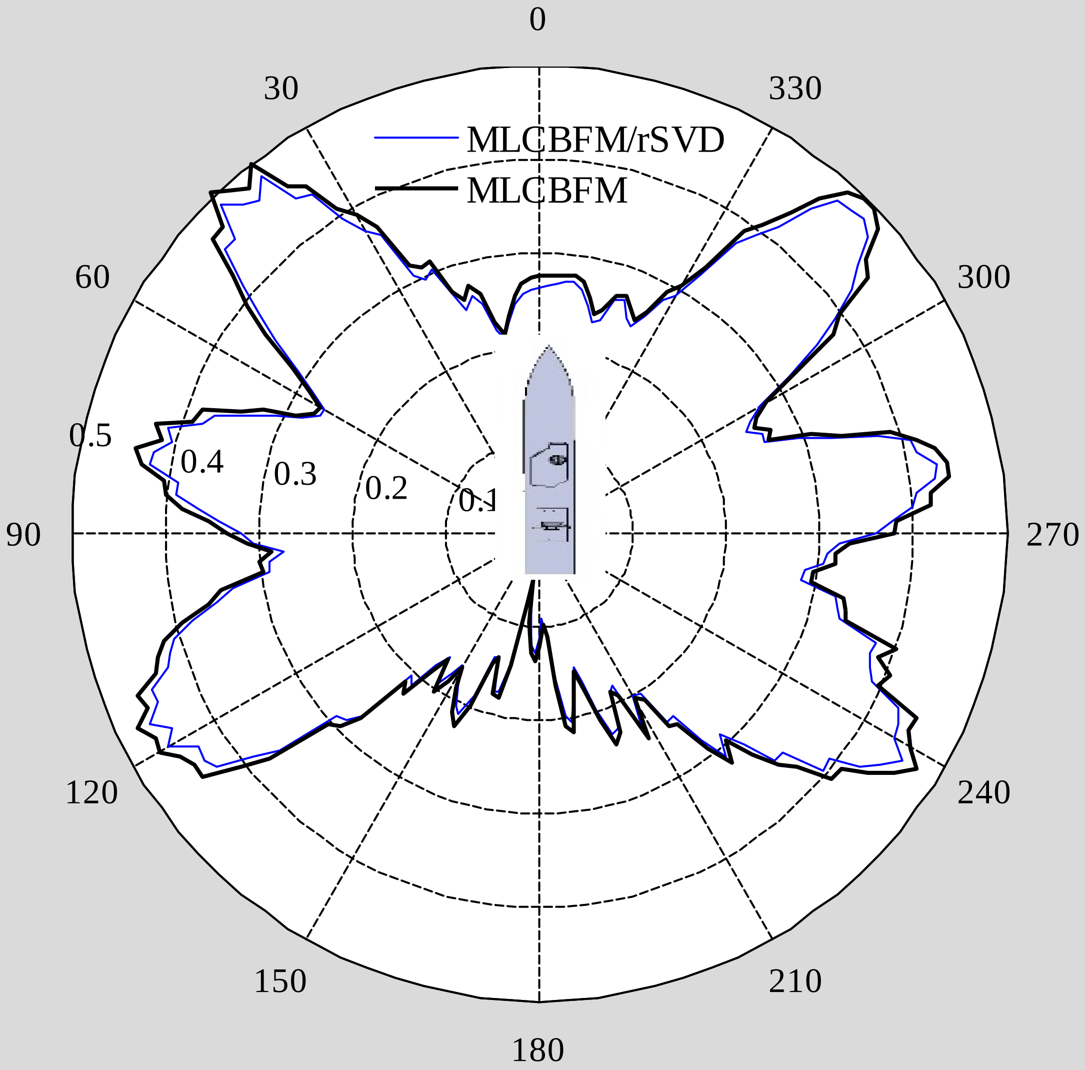


Ship with a monople antenna

Length	Width	Height
156m	21m	28m

❖ The operating frequency is chosen to be 100 MHz and it requires 74,0262, unknowns to discretize the target.

Numerical Result-Radiation Problem



- ❖ 32 Processors

- ❖ 32 Blocks on the first-level
- ❖ 128 Blocks on the second-level
- ❖ Solve Time: 40 Mins
- ❖ Peak Memory: 60GB(approximately)
- ❖ The total CPU time is reduced from 78 minutes to 46 minutes after combining MLCBFM with rSVD.

Conclusion

The rSVD is introduced in this work--for the first time in the context of CEM--and combined with the MLCBFM to solve electromagnetic scattering problems using a domain decomposition approach. The proposed algorithm helps to efficiently derive the CBFs as well as to generate the reduced matrix when using large blocks. Numerical results for a statistical study of the problem of scattering from electrically large random rough surfaces are presented to show that the computation time is significantly reduced, in comparison to that needed in the conventional MLCBFM or Hybrid CBFM/ACA/UV, when the proposed algorithm is employed to solve large problems.